# 06 SOLVING ARHITMETIC PROBLEMS

## GUESSING THE CALENDAR SUM

You have to propose to somebody to enclose 16 dates of a calendar in a 4 x 4 square.

Now, you will write on a paper or on the board a special number (you will know at the end of this explanation).

Now, he or she has to choose a number from the 4 x 4 square, circle it and cross out all the numbers which are in the row and column of this number. And then do same until using all the possibilities.

Mayo 2015

| Domingo | Lunes | Martes | Miércoles | Jueves | Viernes | Sábado |
|---|---|---|---|---|---|---|
| | | | | | 1 | 2 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | | | | | | |

The final result will be four circled dates.

Well, the special number you had written is the sum of these numbers.

### ***Finding out the trick***

The sum will be always $4 \cdot a + 50 - 2$; where 'a' is the first number of the initial square, that is, before choosing any number.

In the example, $4 \cdot 6 + 50 - 2 = 72$

What is the reason?

Try to discover it by using the algebraic language and the distribution of dates in a calendar.

## 1. DISTRIBUTION PROBLEMS

### DIRECTLY PROPORTIONAL DISTRIBUTION

This consists of distributing a quantity '*N* in parts that are directly proportional to some known quantities *a, b, c...*

#### *How to solve these problems*

*1.* Calculate the part of ***N*** that corresponds to a unit

$$k = \frac{N}{a+b+c+...}$$

2. Calculate the proportional part for *a, b, c,...*

### INVERSELY PROPORTIONAL DISTRIBUTION

This consists of distributing a quantity *N* in parts that are inversely proportional to some known quantities *a, b, c...*

#### *How to solve these problems*

You have to distribute N in direct proportion to $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, ...$

#### *By the constant of inverse proportion*

K = x · a = y · b = z · c... So $\frac{k}{a}+\frac{k}{b}+\frac{k}{c}=x+y+z=Total$. We calculate *k* and through it the proportional parts for each one.

### FILLING PROBLEMS

#### *Filling without drain problems*

In these problems we are asked how long it takes two or several taps to fill a tank.

#### *How to solve it?*

1. Calculate the part of the tank that each tap fills in an hour.
2. Calculate the part of the tank that all the taps fill at the same time in an hour.
3. Calculate the time it takes to fill the tank.

### TAPS WITH DRAIN PROBLEMS

Now there is an open drain or a leak.

#### *How to solve it?*

1. Calculate the part of the tank that fills each tap and the part that the drain empties in an hour.
2. Calculate the part of the tank that is filled or emptied in an hour. You have to subtract the emptied part from the filled part.
3. Calculate the time to fill the tank.

## 2. MIXING PROBLEMS

In these problems we are asked the average price at which you have to sell a mixture of several substances, knowing the quantities and prices of each substance.

***How to solve it?***

If mixed $m_1$ kg of a substance *A* and $m_2$ of another *B*, which are priced $P_1$ and $P_2$ respectively, we get a mix of '$m_1 + m_2$'kg at an average price *P*.

1. We can write the data in a table like this:

| | Substance A | Substance B | Mixture |
|---|---|---|---|
| Mass (kg) | $m_1$ | $m_2$ | $m_1 + m_2$ |
| Price | $P_1$ | $P_2$ | P |
| Money | $m_1 P_1 + m_2 P_2 =$ | | $(m_1 + m_2)$ P |

2. Calculate the average price of the mixture: $P = \dfrac{m_1P_1 + m_2P_2}{m_1 + m_2}$

**ALLOY PROBLEMS**

An alloy is a mixture of two or more metals.

Alloy law: Relationship between the weight of fine metal and the total quantities.

***How to solve it?***

Problems are solved in the same way as the problems of mixing, taking into account that the law of the alloy equals the price of the mixture.

$$L = \frac{m_1L_1 + m_2L_2}{m_1 + m_2}$$

## 3. MOVING PROBLEMS AND CLOCK PROBLEMS

**MOVING PROBLEMS IN THE OPPOSITE DIRECTION**

***How to solve them?***

If two objects move in the opposite direction, the velocity with which one approaches is the sum of the velocities of the moving objects.

Step 1. Velocities are added.

Step 2. The time is calculated.

**MOVING PROBLEMS IN THE SAME DIRECTION**

If two objects are moving in the same direction, the speed with which one approaches another is the difference of the velocities of the moving objects.

**CLOCK PROBLEMS**

Clock problems consist of calculating the angle formed by the hands of a clock at a certain time. We must consider that:

The clock face is divided into 12 central angles of 30º each.

The hour hand travels at an angle of 30º per hour while the minute hand runs an angle of 360º.

To solve these problems it is useful to draw a clock with the indicated hour and deduce the angle of separation as the above rules.

**PROBLEMS AND EXERCISES**

## *1. Distribution problems*

**1.** €9900 is going to be dealt in inversely proportional parts to the 3 first classified in a bike race. Calculate the money that belongs to each one.

**2.** €6875 is going to be dealt in inversely proportional parts to the 3 first classified in a bike race. Calculate what belongs to each one.

**3.** a) Share €40 in direct proportion to 3 and 5.
b) Share in inverse proportion.

**4.** a) Share €120 in direct proportion to 3 and 7
b) Share in inverse proportion.

**5.** a) Share €143 in direct proportion to 2, 4 and 5
b) Share in inverse proportion.

**6.** The Administration gives €15000 to help 3 families and the amount of economic aid is in inverse proportion to their incomes. What would the quantity for each family be if family A had €2000 monthly; family B had €2400 and family C had €3000?

**7.** The Administration gives €12000 to help 3 families and the amount of economic aid is in inverse proportion to their incomes. What would the quantity for each family be if family A had €1000 monthly; family B had €1200 and family C had €1500?

**8.** Ann, Catherin and Clair invest 2, 5 and 7 thousand euros, respectively, in a business. If the business produces €742 a week.
a) What fraction of benefits would be for each?
b) How much money would be for each?

**9.** Three shareholders invest 2, 8 and 10 thousand euros, respectively, in a business. They obtained 3.6 thousand in profits one year later. How much would each of the shareholders earn?

**10.** Paving a street has cost €1 800. 4 families —A, B, C y D— are going to pay the expenses in a direct proportion to the frontage. The A house is 5 m long, B is 7.5 m, C is 12.2 m and D is 10.2 m. How much does each family have to pay?

**11.** Three shareholders invest in a business. The first contributes with 1/3 of the capital. The second with 2/5 and the third with the remainder.
Some time later they have to share €30 000 of benefits. How much would it be for each?

## *2. Filling problems*

**12.** A tap takes 3 hours to fill a water tank, and another tap takes 4 hours to fill a tank of the same size. How long will it take to fill a tank of the same size if both taps are distributing water together into the same tank?

**13.** A tap *A* fills a pool in 20 hours, a tap *B* fills the same pool in 25 hours. By law we must always keep open a drain that empties the pool in 100 hours.
How long does it take to fill the pool with both taps open at once?

## *3. Mixing problems*

**14.** We bought 1000 grams of mixed nuts at 10€/Kg. We know that one of the varieties of nuts costs 6€/Kg and there are 600 g in the mixture. How much total does the other variety cost?

**15.** We bought 10 kg of mixed nuts at 10€/kg. We know that one of the varieties of nuts cost 6€/Kg and there are 6 kg in the mixture. How much total euro does the other variety cost?

### *4. Alloys problems*

### *5. Moving problems*

**16.** A truck leaves a city at a speed of 40 mph. An hour later, a car leaves the same city and travels in the same direction at a speed of 60 mph. a) How many hours after leaving the city will the car reach the truck? b) What is the distance from the city where the car will reach the truck?

**17.** Jorge leaves Cáceres going to Mérida in his car with a velocity of 70Km/h and Pablo leaves Casar de Cáceres (12.5 Km to Cáceres) going to Mérida with a velocity of 80 Km/h. If the distance from Mérida to Cáceres is 75 km.

a) At what time are they going to meet?

b) Will Jorge reach Pablo before arriving in Mérida?

(Notice you have to cross Cáceres from Casar de Cáceres to go to Mérida)

**18.** María leaves Cáceres going to Mérida in his car with a velocity of 70Km/h and Lucía leaves Casar de Cáceres (12.5 Km to Cáceres) to Mérida with a velocity of 120 Km/h. If the distance from Mérida to Cáceres is 75 km.

a) At what time are they going to meet?

b) Will María reach Lucía before arriving in Mérida?

(Notice you have to cross Cáceres from Casar de Cáceres to go to Mérida)

### *6. Clock problems*